

A DLA model for Turbulence

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Abstract

A connection between fractal dimensions of "turbulent facets", and fractal dimensions in DLA is shown.

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In a number of articles (see e.g. [1], [2], and later [9]) the fractal dimensions of "turbulent facets" were considered. Among them is the fractal dimension of the interface surface between the potential flow region and the turbulent flow region [9]. Such an interface exists in most classical turbulent flows such as axisymmetric jets, plane wakes and boundary layer turbulence. The fractal dimensions Sreenivasan et al. [9] measured are between 2.3-2.4 with an error of 0.04 (see table-1 for results in 1-D and 2-D slicing). Dimension was measured in the interval between the Kolmogorov scale up to approximately the scale of large eddies (above this scale fractal dimension was found to be not well defined). We claim that the dimension of those interfaces are close and in some flows similar (within the error limit) to the dimension of the interface generated in the DLA model.

It is well-known that the problem of diffusion-limited aggregation (DLA) is useful for describing various phenomena such as viscous fingering [6] and electrical discharge [5] and others [3]. This model provides a solution to the Laplace equation [7]:

$$\Delta u = 0 \tag{1}$$

under the following boundary conditions: $u = \text{const}$ on some far "rigid" boundary and $u = 0$ on a moving boundary F , which moves with the velocity v proportional to the derivative of u in the direction n normal to F plus a

random perturbation p . Thus, for a constant k (which depend on the specific problem in hand, but does not alter the results that concerns us):

$$v_n = kn \cdot \nabla u + p \quad (2)$$

The surface F becomes a thin fractal under the above equations with dimension of 1.70 for the 2-dimensional case and 2.41 in the 3-dimensional case measured by 2-D slicing [4].

Dimensional simialarity does not indicate what kind of mechanism or equations generated this type of surface how ever the equations of potential flow outside the turbulent region and their boundary conditions are quite the same as the ones the DLA model should solve. In this region the velocity field is given by:

$$v = \nabla u \quad (3)$$

since in a liquid the velocity field must obey the continuity equation of the form:

$$\text{div} v = 0 \quad (4)$$

we obtain equation (1). The normal component of the velocity field must certainly be zero on the surface of the vessel containing the liquid, thus on this surface u must be constant sutisfying the first boundary condition. The boundary surface between turbulent and potential flow moves very rapidly [8] and thus the component of the velocity normal to the boundary is much larger than the component of the velocity tangential to the boundary. Therefore this surface may be regarded to a very good degree of approximation as a potential surface too were the value of the potential may be taken conveniently to zero. Thus satisfying the second boundary condition. The velocity of the boundary is of the form (2) with $k = 1$ and p is do to random deviation from pure potential flows. The deviations of the flow from pure potential flow can be treated as a deterministic chaos effect but it also can be taken as a random perturbation due to complexity and smallness. We exclude the influence of large eddies since our description is implied only up to the scale of this eddies.

We thus conclude that the DLA model describes the potential flow near a turbulent boundary, since it satisfies the same equations with the same boundary conditions. The fractal dimension of the interface surface generated by this model is in excellent agreement with experiment. Notice that

we do not intend to explain the phenomena of turbulence, only to offer a simple model for a specific facet of this complex phenomena, which apparently works very well.

Fractal dimension of interface

Flow	2-D slicing	1-D slicing
Boundary layer	2.38	2.40
Axisymmetric jet	2.33	2.32
Plane wake	—	2.37
Mixing layer	—	2.40

Table 1. Summary of the fractal dimensions of the turbulent/ non-turbulent interface in several classical flows [9] .

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